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Adaptive Modal Control of Large Flexible Spacecraft

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A recently developed strategy for adaptive sampled-data control of distributed parameter systems based on a plant modal expansion description and modal simultaneous identification and regulation algorithms is presented with frequent reference to the annular momentum control device test example. The requirement of observation spillover reduction, which is especially crucial to the proposed adaptive control strategy, is addressed.

Nomenclature

a,b	= modal differential equation parameters
A	= number of actuators
C	= number of modes controlled
d	= distributed parameter system deflection
d_R	= reduced-order modal model deflection
$\hat{d}$	= approximate reduced-order model deflection
e	= identification error
f	= actuator forces
F	= modal forces
M	= number of modes in approximate expansion
N	= number of modes in accurate expansion
s	= spatial variable
S	= number of sensors
t	= time
T	= sample period
W	= modal amplitudes
α,β	= modal difference equation parameters
γ,δ	= modal controller parameters
Γ	= AMCD spin rate
θ	= ring particle angle with respect to reference particle
λ	= desired discrete characteristic equation coefficients
μ,ρ	= adaptive identifier step-size weights
σ	= actuator reference frame angles
ϕ	= partial differential equation expansion spatial eigenvectors
ω	= oscillatory modal amplitude time frequency
Ω	= set of distributed system particles

Introduction

THE control of large flexible spacecraft has become an active research and development topic, as demonstrated by a recent survey.[1] Adaptive control of distributed parameter systems (DPS's) is also an emerging research concern.[2-5] Since large flexible spacecraft are acknowledged to be described by partial differential equations with uncertain, i.e., a priori indeterminable, parameters, such structures, requiring increasingly stringent shape and attitude regulation, are prime candidates for appliction of DPS adaptive control strategies.[6]

The objective of the present work is to provide a real-time simultaneous identification and control strategy applicable to DPS's in general and large flexible spacecraft in particular. The real-time computation objective prompts the use of modal expansion descriptions of DPS's to permit some parallel computation. The constantly activated adaptability of simultaneous identification and control is needed, for example, for adequate control of poorly behaved DPS's, such as very lightly damped, large flexible spacecraft during inadequately predictable plant parameter changes due to operating condition variations. Toward this goal, this paper improves a previously proposed strategy[4] with further attention to a step-by-step adaptive controller development procedure and its consequences. The original idea[4] was to combine a truncated modal expansion description used in flexible spacecraft control[1] with a simultaneous identification and control (also termed self-tuning[7]) adaption strategy to regulate the lumped-parameter modal amplitude descriptions. The procedure of the next section also mentions the possibility of direct rather than indirect modal control parameter adaption, such as via model reference adaptive control.[8] The outlined procedure also reacts further to the special problems spillover[9] creates for an adaptive implementation. This strategy is followed in the third section to develop an adaptive regulator of the linearized, out-of-plane deflection of a spinning annular momentum control device (AMCD), a candidate for large flexible spacecraft control.[4,10,11] Simulations of this appliction are presented in the fourth section. This example is also used in the fifth section to illustrate the difficult modal observation spillover reduction problem. An enlarged framework for adaptive control of DPS's, especially flexible spacecraft, is overviewed in the conclusion. Unfortunately, achievement of the grandiose objective stated at the start of this paragraph is only (possibly) begun in this paper.

A DPS Adaptive Control Strategy

For a DPS describable by

$$d(s,t)=\sum_{j=0}^{\infty} W_j(t)\phi_j(s) \tag{1}$$

where d is a vector-valued function of spatial location s and time t, e.g., displacement for flexible spacecraft, $\phi_j(s)$ is an orthogonal expansion basis of shape eigenfunctions, and $W_j(t)$ is the amplitude of the jth shape function at time t, an adaptive control strategy has been proposed.[4] Under the assumption that the amplitudes obey uncoupled, linear, ordinary differential equations of known order, but with unknown coefficients, e.g.,

$$\frac{d^n}{dt^n}W_j(t)=\sum_{i=1}^{n}\left[a_{ji}\frac{d^{n-i}}{dt^{n-i}}W_j(t)+b_{ji}\frac{d^{n-i}}{dt^{n-i}}F_j(t)\right] \tag{2}$$

where $F_j(t)$ are the modal forces, then this strategy[4] combines a truncation of Eq. (1) with a modal self-tuning[7] (or simultaneous identification and control) adaptive control algorithm. The modal forces F_j in Eq. (2) are given by

$$F_j(t)=\int_{s\in\Omega}\phi_j(s)f(s,t)\,ds \tag{3}$$

where Ω is the set of all system particles, and $f(s,t)$ is the applied spatially distributed force. The espoused approach can control the DPS of Eq. (1) despite a lack of knowledge in the a_{ji} and b_{ji} of Eq. (2), which can accommodate inaccurate a priori expansion modeling or variability in the a_{ji} and b_{ji} due

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to changes in the operating conditions. This approach, to be outlined later relies on 1) the prespecification of the ϕ_j in Eq. (1) yielding uncoupled Eq. (2), and 2) the reasonableness of Eq. (1) after truncation. The second assumption will be extensively examined in the latter sections of this paper.

The adaptive modal control strategy[4] can be divided into two stages: one prior to system operation and the other on-line. The recommended steps of preactivation analysis are:

1) Determine expansion basis ϕ_j in Eq. (1).

2) Select finite expansion upper limit to approximate Eq. (1).

3) Specify sensor locations and relate distributed measurements of $d(s,t)$ to the modal amplitudes $W_j(t)$ in Eq. (2) via reversal of Eq. (1).

4) Determine actuator distribution and formulate effect on modal forces $F_j(t)$ in Eq. (2) via Eq. (3).

5) Establish modal control objectives.

The recommended steps of real-time, adapting, sampled-data control formation are:

6) Apply previously calculated actuation forces and sense $d(s,t)$ in Eq. (1) at sample instant.

7) Process sensor data to estimate modal amplitudes $W_j(t)$ in Eq. (2).

8) Select the modes requiring control.

9) Process applied forces $f(s,t)$ via Eq. (3) to determine achieved modal forces $F_j(t)$.

10) Improve the identification of the discretization of Eq. (2).

11) Design modal controllers on-line with current parameter estimates to meet modal performance objectives.

12) Convert desired modal control to actual actuator commands.

13) Repeat steps 6-12 at the next sample instant.

The simultaneous identification and control strategy of steps 10 and 11 could be replaced by a single step improving the controller parameters by direct lumped-parameter plant adaptive control.[12-14] The espoused indirect strategy is more intuitive and currently more flexible in terms of control objectives though more restrictive in terms of the general need for plant parameter identifiability,[15,16] as manifested in adequate model complexity and sufficiently exciting input requirements.

As outlined, the modal controllers rely only on past data to determine the present control action. This allows the considerable (despite the possible parallel execution of steps 10 and 11) computation to be done during the full sample period. Steps 6-12 differ slightly from their earlier description[4] due to a fuller appreciation of the demands of steps 7 and 9, especially 7, which are discussed in later sections. Further, detailed comments on each stage of this step-by-step procedure are available.[4] The next section illustrates this strategy by application to adaptive regulation of the small, linearized, out-of-plane deflection of a large, flexible AMCD.

AMCD Application

Large momentum vectors resulting in the rotation of large space structures can be created smoothly by a solar-powered (and therefore, effectively non-depletable), dual momentum vector configuration of two counterrotating AMCD's magnetically attached to the space structure.[4,10,11] The AMCD components of these attitude control devices will probably be as large in diameter as possible in order to maximize their momentum/mass ratio. Therefore, such AMCD's would behave like lariats with translation, rotation, and deformation modes of disturbance from their nominal planar spinning configuration.

The step-by-step procedure of the preceding section will be followed in designing an adaptive modal controller of such an AMCD.

1) *The boundary conditions of ring closure* permit the use of a Fourier series to describe the linearized, out-of-spin-plane deformation of the AMCD

$$d(\theta,t)=\sum_{j=0}^{\infty}[W_{cj}(t)\cos(j\theta)+W_{sj}(t)\sin(j\theta)] \tag{4}$$

which is of the form of Eq. (1) with the angle θ, measured around the AMCD ring from a reference ($\theta=0$) particle to the location in question, as the single spatial variable s. This sinusoidal basis is also the eigenvector basis for this out-of-plane motion of a homogeneous ring.[17]

2) *Assuming that the higher spatial frequency deformations* will exhibit lower amplitudes, Eq. (4) can be truncated with arbitrary accuracy as

$$d(\theta,t)=\sum_{j=0}^{N}[W_{cj}(t)\cos(j\theta)+W_{sj}(t)\sin(j\theta)] \tag{5}$$

This limit N may permit accurate approximation of Eq. (4) but be an infeasible limit in terms of controller computations. If a further reduction is necessary, Eq. (5) and therefore Eq. (4) [and in effect, Eq. (1)] will be roughly approximated by either

$$d_R(\theta,t)\triangleq\sum_{j}^{M\text{ of }[0,N]}[W_{cj}(t)\cos(j\theta)+W_{sj}(t)\sin(j\theta)] \tag{6}$$

where

$$\sum_{j}^{M\text{ of }[0,N]}$$

signifies a summation over the index j, where j is any $M+1$ entries of the set $\{0,1,2,...,N\}$, or

$$\hat{d}(\theta,t)\triangleq\sum_{j}^{M\text{ of }[0,N]}[\hat{W}_{cj}(t)\cos(j\theta)+\hat{W}_{sj}(t)\sin(j\theta)] \tag{7}$$

where the $\hat{W}_k$ are not necessarily the corresponding W_k in Eqs. (6) and (5) due to the selection of $\hat{W}$ to best fit $\hat{d}$ to d given $2M+1$ point measurements of d, e.g., as in the Galerkin approach.[1]

3) *Assume* S *sensor measurements* of ring particle deflections $d(\theta_i,t)$ at $i=1,2,...,S$ can be processed simultaneously. These measurements can be decomposed into modal amplitudes W_j by multiple concatenation of Eq. (6) [or Eq. (7)] with

$$\sum_{j}^{M\text{ of }[0,N]}\quad\text{replaced by}\sum_{j=0}^{M}\text{, i.e.}$$

for the lower modes, as

$$\begin{bmatrix}d_R(\theta_1,t)\\ d_R(\theta_2,t)\\ \vdots\\ d_R(\theta_S,t)\end{bmatrix}=\begin{bmatrix}1 & \cos(\theta_1)\ldots\cos(M\theta_1) & \sin(\theta_1)\ldots\sin(M\theta_1)\\ 1 & \cos(\theta_2)\ldots\cos(M\theta_2) & \sin(\theta_2)\ldots\sin(M\theta_2)\\ \vdots & \vdots \quad \vdots & \vdots \quad \vdots\\ 1 & \cos(\theta_S)\ldots\cos(M\theta_S) & \sin(\theta_S)\ldots\sin(M\theta_S)\end{bmatrix}\begin{bmatrix}W_0(t)\\ W_{c1}(t)\\ \vdots\\ W_{cM}(t)\\ W_{s1}(t)\\ \vdots\\ W_{sM}(t)\end{bmatrix} \tag{8}$$

With appropriate reindexing of the right side of Eq. (8), any M modal amplitudes composing d_R in Eq. (6) could be written in this matrix form. The θ_i may vary from sample to sample, especially if the sensor(s) is not spinning with the AMCD. Note that Eq. (8) requires measurement of d_R, not d. Obtaining d_R from d requires observation spillover removal, as will be addressed later.

4) *Assume* A *actuators are located* in a reference frame fixed with respect to AMCD spin, e.g., the suggested[11] magnetic "bearing" actuators attached to the spacecraft. From Eq. (3), where $\Omega = \{\theta \mid \theta \in [0,2\pi)\}$, an assumption of point actuation located at σ_i converts the integrals to summations over the set of A actuators. Note that, due to an AMCD spin rate of Γ rad/s relative to the actuator locations, the σ_i must be converted to the ring particle reference frame via

$$\theta_i(t) = \sigma_i(t) - \Gamma t \tag{9}$$

which assumes that the references $\theta = 0$ and $\sigma = 0$ were aligned at $t = 0$. Similar to Eq. (8), for the C modes to be controlled

$$\begin{bmatrix} F_0(t) \\ F_{c1}(t) \\ \vdots \\ F_{cC}(t) \\ F_{s1}(t) \\ \vdots \\ F_{sC}(t) \end{bmatrix} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \cos(\theta_1) & \cos(\theta_2) & \cdots & \cos(\theta_A) \\ \vdots & \vdots & & \vdots \\ \cos(C\theta_1) & \cos(C\theta_2) & \cdots & \cos(C\theta_A) \\ \sin(\theta_1) & \sin(\theta_2) & \cdots & \sin(\theta_A) \\ \vdots & \vdots & & \vdots \\ \sin(C\theta_1) & \sin(C\theta_2) & \cdots & \sin(C\theta_A) \end{bmatrix} \begin{bmatrix} f(\theta_1,t) \\ f(\theta_2,t) \\ \vdots \\ f(\theta_A,t) \end{bmatrix} \tag{10}$$

can be formed.

5) *Ring stabilization* requires mode damping. Satisfactory modal damping can be provided by modal pole placement.

6) *If* d(θ,t) *is provided* by an effectively instantaneously scanning sensor, then any S ring particles can be observed. If the sensors are incorporated with the actuators, then Eq. (9) must be used since different ring particles will be sensed at each sample instant. One other possibility is more frequent sensor interrogation than actuator reactivation, allowing additional signal processing possibilities before control selection.

7) *Solution for the* [W_j] *vector* in Eq. (8) can be achieved by pseudoinversion[18] or by a DFT[19] if the θ_i are equally spaced. Note that if the θ_i are equally spaced such that $\theta_i = 2\pi i/S$ and $S = 2M$, then $M\theta_i = \pi_i$, and the rightmost column of the $S \times (2M+1)$ matrix in Eq. (8) equals zero. Therefore, the column of $\sin(M\theta_i)$ entries should be removed to retain invertibility. In such a case, the Mth mode sine component is unobservable.

8) *If* C<M, *the current strategies* are to control those C modes either with the lowest spatial frequencies or with the greatest modal amplitudes. The assumption leading to Eq. (5) will tend to equate these two classes.

9) *If* A<2C+1 *in Eq. (10)*, then due to the least-squares solution implemented in step 12 of the last sample instant, the desired F_j most likely will not have been achieved, and Eq. (10) must be calculated to determine the forces actually reaching each mode needed in the next step.

10) *For a large lightweight AMCD*, Eq. (2) will be second-order and essentially undamped

$$\frac{d^2}{dt^2}[W_j(t)] + \omega_j^2(\Gamma,t)\,W_j(t) = \left(\frac{1}{M_j}\right)F_j(t) \tag{11}$$

where the M_j denote the effective modal masses,[4] and the Γ and t arguments of ω are intended to evoke the slowly time-varying character of the modal amplitude time frequency due to such operating conditions as spin rate and temperature. Note that the magnitude of the oscillatory initial condition response of Eq. (11) is inversely proportional to ω_j. The notation of Eq. (11) [and subsequently, Eqs. (12-20)] is intended to encompass both W_{cj} and W_{sj}. Note that both W_{cj} and W_{sj} have the same ω_j, i.e., $\omega_{cj} = \omega_{sj}$. Assuming uniform sample intervals of T seconds and constant modal forces over the sample period,

$$W_j(k) = \alpha_{j1}W_j(k-1) + \alpha_{j2}W_j(k-2) + \beta_{j1}F_j(k-1) + \beta_{j2}F_j(k-2) \tag{12}$$

is an exact discretized predictor of the modal amplitude where $\alpha_{j1} = 2\cos(\omega_j T)$, $\alpha_{j2} = -1$, and $\beta_{j1} = \beta_{j2} = (1-\cos\omega_j T)/(M_j\omega_j^2)$. The addition of damping in Eq. (11) will effect the definitions of α and β but not the form of Eq. (12). Note that constant actuator forces f over the sample interval will not generate constant modal forces due to AMCD rotation. This can be combatted by actuator force windowing. If the modal forces vary over the sample period, Eq. (12) becomes an approximation only as accurate as the degree of constancy of $F_j(t)$ over $(k-2)T<t<(k-1)T$ and $(k-1)T<t<kT$. If the structure of Eq. (12) is used for an adaptive identifier or to structure a direct adaptive controller, then uncertainty in ω_j, M_j, and the neglected damping coefficient can be accommodated. The anticipated problem in AMCD control is uncertainty in ω_j. The second-order Eq. (12) can be identified by either of two broad classes of recursive parameter estimators termed[20] prediction error and pseudo linear regression and represented by equation error[21] and output error[22] identifiers, respectively. An equation error formulated identifier for Eq. (12) of the form

$$\begin{bmatrix} \hat{\alpha}_{j1}(k) \\ \hat{\alpha}_{j2}(k) \\ \hat{\beta}_{j1}(k) \\ \hat{\beta}_{j2}(k) \end{bmatrix} = \begin{bmatrix} \hat{\alpha}_{j1}(k-1) \\ \hat{\alpha}_{j2}(k-1) \\ \hat{\beta}_{j1}(k-1) \\ \hat{\beta}_{j2}(k-1) \end{bmatrix} + \frac{e_j(k-1)}{1+\sum_{i=1}^{2}[\mu_{ji}(k-1)W_j^2(k-i-1)+\rho_{ji}(k-1)F_j^2(k-i-1)]} \begin{bmatrix} \mu_{j1}(k-1)W_j(k-2) \\ \mu_{j2}(k-1)W_j(k-3) \\ \rho_{j1}(k-1)F_j(k-2) \\ \rho_{j2}(k-1)F_j(k-3) \end{bmatrix} \tag{13}$$

where

$$e_j(k-1) = W_j(k-1) - \sum_{i=1}^{2} [\hat{\alpha}_{ji}(k-1)\, W_j(k-i-1)$$
$$+ \hat{\beta}_{ji}(k-1) F_j(k-i-1)] \qquad (14)$$

and

$$0<\mu_{ji}(k) \le \mu_{ji}(k-1) < 2 \quad \text{and} \quad 0<\rho_{ji}(k) \le \rho_{ji}(k-1) < 2$$
$$(\forall \quad i,k) \qquad (15)$$

requires exact measurements of sufficiently rich F_j and W_j for consistent identification.

11) *Feedback regulation structures* of second-order dynamic output feedback[4] or equivalently (in the absence of unmeasurable inputs) reconfigured state variable feedback achieved via

$$F_j(k) = \hat{\gamma}_{ji}(k)\, W_j(k-1) + \hat{\gamma}_{j2}(k)\, W_j(k-2)$$
$$+ \hat{\delta}_{j1}(k) F_j(k-1) + \hat{\delta}_{j2}(k) F_j(k-2) \qquad (16)$$

will cause the modal plant-controller characteristic equation to converge to $z^4 + \lambda_{j1} z^3 + \lambda_{j2} z^2 + \lambda_{j3} z + \lambda_{j4}$ if the controller parameters are chosen via

$$\hat{\delta}_{j1}(k) = -\lambda_{ji} - \hat{\alpha}_{j1}(k) \qquad (17)$$

$$\hat{\gamma}_{j1}(k) =$$
$$[\hat{\delta}_{j1}(k)\hat{\alpha}_{j2}(k) - \lambda_{j3} + (\hat{\delta}_{j1}(k)\hat{\alpha}_{j1}(k) - \hat{\alpha}_{j2}(k) - \lambda_{j2})$$
$$\times (\hat{\alpha}_{j1}(k) - \hat{\alpha}_{j2}(k)\hat{\beta}_{j1}(k)/\hat{\beta}_{j2}(k)) + \hat{\lambda}_{j4}\hat{\beta}_{j1}(k)/\hat{\beta}_{j2}(k)]$$
$$\times [\hat{\beta}_{j1}(k)\hat{\alpha}_{j1}(k) + \hat{\beta}_{j2}(k) - \hat{\alpha}_{j2}(k)\hat{\beta}_{j1}^2(k)/\hat{\beta}_{j2}(k)]^{-1} \qquad (18)$$

$$\hat{\delta}_{j2}(k) = \hat{\alpha}_{j1}(k)\hat{\delta}_{j1}(k) - \lambda_{j2} - \hat{\beta}_{j1}(k)\hat{\gamma}_{j1}(k) - \hat{\alpha}_{j2}(k) \qquad (19)$$

$$\hat{\gamma}_{j2}(k) = (\hat{\delta}_{j2}(k)\hat{\alpha}_{j2}(k) - \lambda_{j4})/\hat{\beta}_{j2}(k) \qquad (20)$$

and $\hat{\alpha} \to \alpha$ and $\hat{\beta} \to \beta$. Note that the strictly causal formation of $\hat{\alpha}$ and $\hat{\beta}$ in Eqs. (13-14) used in Eqs. (17-20) to parameterize Eq. (16) permits the assumed predictive control formation. Furthermore, the form of Eq. (16) purposely avoids velocity measurements, which are expected to be difficult to sense for large AMCD's.

12) *Least-squares solution* of Eq. (10) for the $f(\cdot,t)$ is required if $A<2C+1$.

Simulations

Successful simulations of this AMCD example have been reported elsewhere.[4,11,23,24] Consider here an AMCD described by Eq. (5) with $N=4$ and the ω_j in Eq. (11) equal to 0, 2.62, 5.24, 7.85, and 10.47 for $j=0, 1, 2, 3, 4$, respectively. For an AMCD spin rate of $\Gamma = 60$ deg/s, a sample period of $T=0.1$ s, and the same damped pole placement objective for each mode of $\lambda_1 = -1.684$, $\lambda_2 = 1.165$, $\lambda_3 = -0.402$, and $\lambda_4 = 0.0558$, the strategy of the preceding section was applied in the following cases: $N=M=C=4$, $N=M=4>C=2$, and $N=4>M=C=2$. For each situation, the AMCD was given an initial deformation composed of unity modal amplitudes. The adaptive controller was applied, with initial modal frequency estimates $\{\hat{\omega}_j\}_{j=1,\ldots,4} = \{4, 8, 12, 16\}$ converted to initial parameter estimates in Eq. (12) via the discretization formulas, in an attempt to stabilize the ring deflections to zero.

The AMCD simulation consisted of Eq. (10) with C replaced by N, which converted the applied forces f to modal forces F which were assumed constant over the sample interval, the use of the appropriate F in Eq. (12) to update the N modal amplitudes W, and the formation of the deflection d via Eq. (5) for all θ. The controller began with measurements of d for use in Eq. (8), which were supplied by solution of Eq. (5) at the appropriate sensor locations θ. The sensor locations were assumed to be rotating with the AMCD. A special reflective mark on the AMCD and a centrally located, scanning laser detection system can be hypothesized as providing such ring particle deflections. The measured d were used for the d_R in Eq. (8) with $M=4$ or 2 as required. Solution of Eq. (8) provided the W used in Eqs. (13) and (16). The improved parameter estimates provided by Eq. (13) with all $\mu=\rho=1$ were used in Eqs. (17-20) to parameterize Eq. (16). When $M>C$, only the F for the modes of lower spatial frequency were calculated. The past F in Eq. (16) were provided by the previous results of Eq. (16). Step 9 could be bypassed since A was always chosen equal to or greater than $2C+1$. Inversion solution of Eq. (10) provided the applied forces f to the AMCD simulation.

Figure 1 with $N=M=C=4$ illustrates the anticipated effectiveness of the adaptive strategy in recovering from inaccurate estimates of the ω_j and successfully stabilizing the initial AMCD deflection, eventually to zero displacement. The asterisks on the displacement curves mark the measurement points which are fixed in the ring particle reference frame. The arrowheads along the spatial coordinate θ axis show inertially fixed actuator locations with the length of the arrowshaft proportional to the applied force according to the right-hand scale. The plots are drawn in the ring particle reference frame so the actuator locations appear to regress for the progressing ring. The success in Fig. 1 is not universal for $N=M=C$. With only the desired pole locations changed to values nearer the unit circle by $\lambda_1 = -2.314$, $\lambda = 2.429$, $\lambda_3 = -1.176$, and $\lambda_4 = 0.2$, the proposed adaptive regulator fails via an often neglected stall mechanism. For example, if the parameters in Eq. (13) when used in Eqs. (17-20) lead to an unstable modal controller, W_j^2 will become so large in the denominator in Eq. (13) that the parameter estimates are only insignificantly corrected. If this condition persists long enough, all claim of linearity can be abandoned in application, thereby negating claims of eventual identifier convergence. This stall mechanism, a characteristic of simultaneous identification and control not peculiar to just

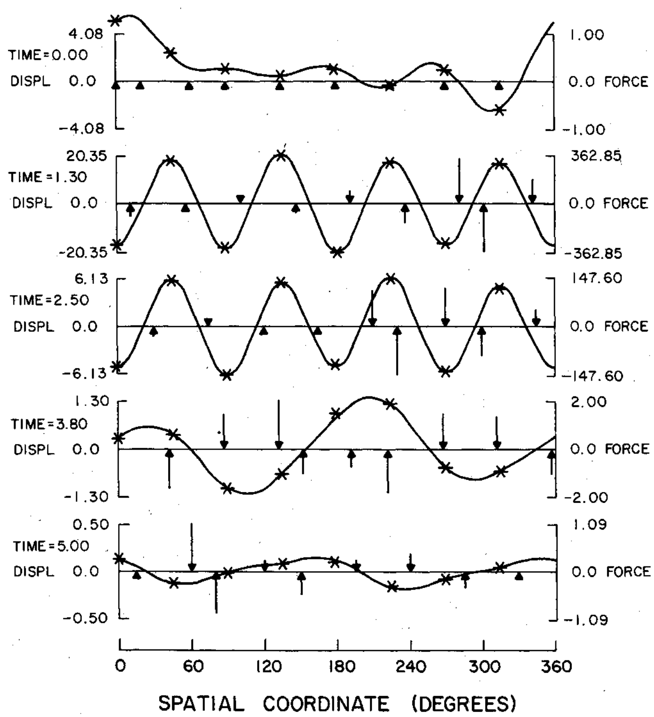

Fig. 1 Adaptive regulation without spillover.

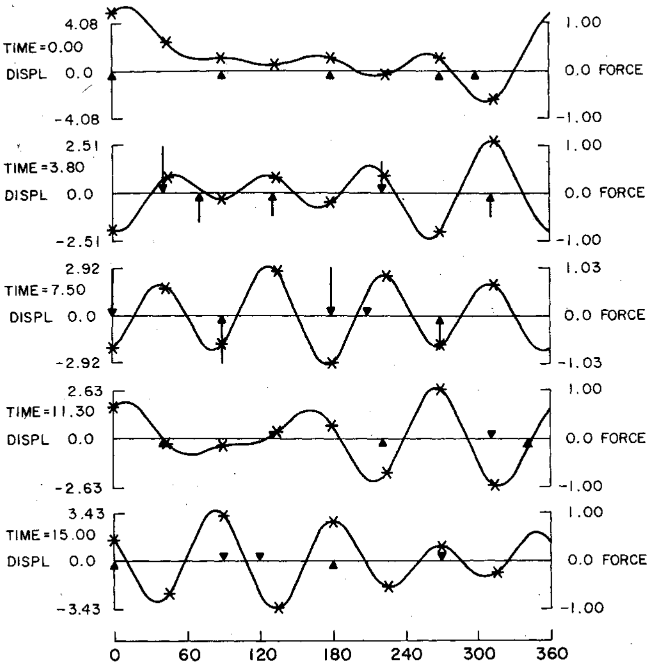

Fig. 2 Adaptive regulation with control spillover.

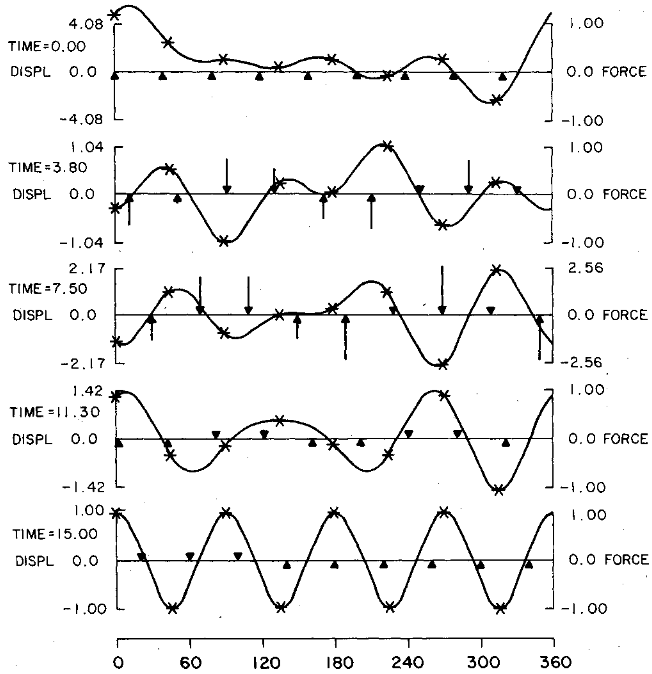

Fig. 3 Adaptive regulation with control spillover.

the proposed adaptive DPS control strategy, occurs in the cited example with the W_{c4} (5 s) $= 4.8 \times 10^4$ and growing. This failure can be attributed to the smaller stability margin of higher modal frequency estimates for low-frequency, light damping objectives. Different initial parameter estimates nearer the actual modal frequencies can avoid the stall of this particular example. Also, the sufficient excitation requirements of simultaneous identification and control[7] are almost surely not met in this regulation example, thereby encouraging an expectation of failure. However, this pathological case does emphasize a need for an understanding (currently nonexistent even in lumped-parameter system adaptive control) of closed-loop singularity migration during simultaneous identification and control.

Figure 2 illustrates the boundedness of the AMCD deflection achieved in adapting from inaccurate ω_j prespecification with $N=M=4>C=2$. The visible higher spatial frequency remnants at $t=15$ s are due to the choice to control only the lower spatial frequency components of d, which do decay to zero. Asymptotic regulation of the full deflection is not achieved despite convergent parameter identification in the satisfactorily complex model due to control spillover arising from use of a restricted complexity controller.[9] For lumped-parameter systems, this limited success of adaptive control when $N=M>C$ has also been documented elsewhere.[25] For $A>2C+1$, i.e., an over-sufficient number of actuators for solution of Eq. (10), pseudoinverse solution of Eq. (10) provides a minimum control f energy solution, which can be expected to reduce the deleterious control spillover. Figure 3 with $A=9$ for $C=2$ vs Fig. 2, where $A=5$ for $C=2$, clearly demonstrates this effect.

The possible failure of the proposed adaptive control strategy when $N>M$ is documented in Fig. 4, where $N=4>M=C=2$. The identifier of Eq. (13) is unsuccessful in converging on the actual α and β values due to the use of d and not d_R in Eq. (8). Since d is used in Eq. (8) and a least-squares solution is used to determine the W, then actually Eq. (7) and not Eq. (6) is applicable. Figure 4, therefore, shows the failure of attempting to use a modal control strategy when approximately matching the full AMCD behavior with a reduced number of modes. For this example, even if the α and β were successfully identified, using the $\hat{W}_j$ and not the W_j in Eq. (16) leads to instability. Clearly, a mixture of full behavior estimation and reduced-order control strategies, though commonly pursued in practice, is only valid if the modes omitted from the model contribute negligibly to the total behavior. Due to the control spillover reaching the unmodeled modes, even in the event of no initial energy in these modes, this negligibility cannot be assumed. The difficulty adaptive control experiences due to $N>M$ has also been noted in the lumped-parameter system case.[26] The next section considers a signal processing strategy to combat this possibly fatal problem by filtering the d of Eq. (5) to provide the d_R of Eqs. (6-8).

Observation Spillover Reduction

Inexact sampled W_j are provided to the modal identifiers due to two sources of error: 1) aliasing, both in time and spatial frequency domains, due to discrete measurements of $d(s,t)$, and 2) reduced-order modeling inaccuracy due to $N>M$. However, based on the characteristic of flexible spacecraft that higher modal frequencies, e.g., the higher frequency ϕ_j for the AMCD in Eq. (4) as j becomes large, have correspondingly higher frequency ϕ_j for the AMCD in Eq. (4) as j becomes large, have correspondingly higher modal amplitude time frequencies, i.e., $\omega_i>\omega_j$ for $i>j$, a strategy for extracting d_R in Eq. (6) from measurements of d in Eq. (5) has been postulated.[24] This approach assumes: a) satisfactory time-based sampling to avoid aliasing of modal frequencies up to ω_N, i.e., $T<\pi/\omega_N$; b) a sensor system capable of interrogating any ring particle at any sample instant; c) band-limiting knowledge of the M sought ω_i, e.g., for the lowest M consecutive ω_i specification of a frequency comfortably between ω_M and ω_{M+1}; and d) frequency-limited spectra for the F_j leading to separable W_j spectra at desired cutoff points.

For a lowest M frequency approximation, implicit in Eq. (8), the following strategy appears reasonable. Assuming equally spaced sensor measurements, in both space and time, and $S=2M$, as in point 7 of the preceding section, a DFT could be used to solve Eq. (8); however, d_R and not d must be available. If the same ring particle can be measured for deflection at successive sample instants despite AMCD rotation, then the sequence $\{d(\theta_i,kT)\}$ over k for a particular

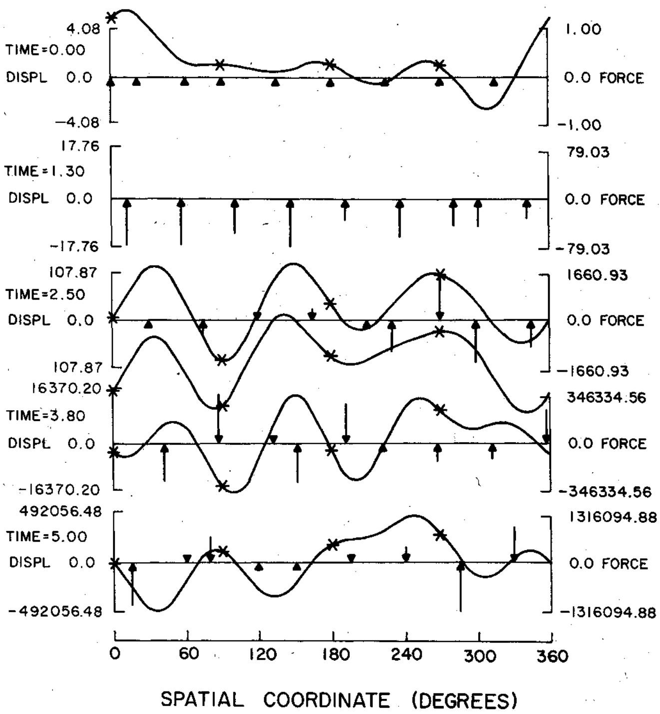

Fig. 4 Attempted adaptive regulation with control and observation spillover.

i, which is proportional from Eq. (5) to a fixed weight sum of the $W_j(kT)$, can be low-pass filtered (LPF) between ω_M and ω_{M+1} removing high time, and therefore, also spatial, frequency components leaving $\{d_R(\theta_i, kT)\}$. For each time t, composition of Eq. (8) is now possible.

The necessary assumptions preceding this particular strategy description are quite restrictive. Due to the true form of Eq. (1) and the possibly tremendous magnitude of N and therefore ω_N, assumption a is mathematically impossible and only marginally practical. The limitation of assumption b is that the sensors cannot be colocated with the nonrotating actuators. Even if AMCD spin could be accommodated for DFT purposes, the same ring particle could not be sensed at sequential sample instants, disallowing the benefit of LPF. Conversely, always measuring the same ring particles could, in certain cases, lengthen the lag time before reaction of the control system to slowly propagating localized disturbances. Assumption c may be reasonable for discrete sinusoidal spectra, but since Eq. (11) is forced by a nonlinearly generated, nonstationary (during adaptation) signal in violation of assumption d, the region of nonoverlap between ω_M and ω_{M+1} becomes so small (if not nonexistent) as to require highly refined a priori knowledge. LPF phase distortion must also be assumed negligible. Clearly, some higher spatial frequencies will be rejected by the LPF, but due to dissatisfaction of the assumptions pointing to unsatisfactory spectrum separation, d_R will be inexactly obtained. These reservations are even more severe for more complex filtering schemes, such as the effective comb filtering suggested earlier[6] to be achieved via phase locked loops.

One alternative[6] satisfying the c and d requirements for filtering d_R from d is identification of the free AMCD response. However, this approach does not meet the stated simultaneous identification and control objective. Using a modified gain scheduling concept[27] to provide a fixed robust control during identification phases is closer to the simultaneous identification and control objective (and may avoid the stall mechanism noted in the previous section), but does spread the W_j spectra, again severely limiting the benefits of the LPF. Another seemingly applicable concept is that of adaptive orthogonal filtering.[28] This idea is incorporated by appending to Eqs. (8) and (13) additional uncontrolled "modes" intended to absorb the spillover effects. These modes would require time-varying dynamic descriptions, meaning that the μ and ρ used in estimating their difference equation parameters would need to be significantly larger than those used in estimating the time-invariant modal difference equation parameters. The compensatory ability of these additional model modes seems limited due to the assumptions necessary in adaptive orthogonal filter development. Therefore, none of these suggestions appears wholly satisfactory. The conclusion is that currently the observation spillover problem remains unsolved, yet requires resolution for broad applicability of the proposed adaptive modal control scheme for DPS's.

Conclusion

This paper begins with revision of a previously originated strategy[4] for adaptive modal control of distributed parameter systems (DPS's) and concludes with the confrontation of the spillover problem, which is extremely severe. In support of the simulation evidence provided, it has been proven elsewhere[5] that in the absence of observation spillover and with the use of the eigenvector expansion, adaptive modal control of DPS's is as viable as lumped-parameter system indirect adaptive control. It can also be shown[6] why a stable simultaneous identification and control scheme similar to that imbedded in the annular momentum control device example fails in the presence of observation spillover or nonorthogonal expansion. With the necessity of a reduced-order model ($N>M$), the goal of globally stable adaptive DPS controller convergence appears too stringent. Work is in progress to relate observation spillover bounds to parameter identification bounds. Such efforts are directed toward delineating the detrimental influence of observation spillover and the possibility of allowable behavior despite its presence rather than toward its removal.

Suggestions have also been forwarded[29] to remedy the difficulty of ϕ_j selection. For general spacecraft, the fundamental assumption of eigenvector availability for Eq. (1) is overly optimistic. Slight inhomogeneities in the annular momentom control device can lead to significant coupling of the Fourier expansion "modes."[30] Such coupling, if incorporated in the system model, disallows the parallel computation objective for the identifier and "modal" controller. The specter of the necessity to recursively estimate both the basis functions and their associated dynamics raises questions far beyond the current scope of these efforts. However, these are issues that ultimately must be addressed.

The development of an adaptive controller applicable to DPS's requires examination of both indirect and direct adaptive control concepts in a necessarily reduced-order model format. Both approaches are susceptible to spillover degradation. A judicious mixture of robust control, gain-scheduling, on-line vs off-line identification, specific "optimal" objectives vs simpler damping requirements, and local vs global convergence behavior will be required in subsequent efforts. Further research, as in any emerging field, will better identify the weaknesses and strengths of proposed approaches to adaptive control of DPS's and uncover additional concerns requiring further original developments.

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